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By modelling the coronal structures by "slowly" evolving Double–Beltrami two–fluid equilibria (created by the interaction of the magnetic and velocity fields), the conditions for catastrophic transformations of the original state are derived. It is shown that, at the transition, much of the magnetic energy of the original state is converted to the the flow kinetic energy.

Subject headings: Sun: flares — Sun: corona —- Sun: magnetic fields

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I. INTRODUCTION

The latest TRACE and SOHO/EIT observations have brought the solar corona into sharp focus. The observations reveal: 1) the structures that constitute the solar corona are in constant motion; they are full of fast–moving gases, and are heated primarily at their footpoints (base) very close to the solar surface. The heating occurs in few minutes in the first ten to twenty thousand kilometers above the surface, i.e, in a rather small fraction of the bright part of the anchored structure. In direct contradiction to the predictions of some theories, the heating is neither uniform (throughout the loops) nor does it happen preferentially near the top. A direct quote sums up the situation aptly: "Moreover, not only heat is deposited low down, but the gas is often actually thrust upward very rapidly. It does not merely 'evaporate' into the coronal structures, it is often actually thrown up there. Exactly how that happens is still a puzzle" [1], 2) the loops are composed of clusters of filamentary structures which are not, as believed before, static bodies supported by interior gas pressure and heated along their lengths [2]. They fill and drain so quickly that the gas in them must be moving nearly ballistically (see latest TRACE news and e.g. [1]) along the substructures, rather than being "quiescently heated". From a detailed study of the loops with different characteristic parameters one concludes that the heating process is quite non–uniform [3].

Transient brightenings, with their associated flows of cool and hot material, are also a very common phenomenon in the TRACE movies. These relatively fast (violent) happenings vary from small events in the quiet Sun to major flares in active regions; brightenings which are more than 10^5 km apart often occur within the same exposure that typically lasts for 10 to 30 s [1]. This kind of a coincidence in the events at distant locations is suggestive of fast particle beams propagation along separate magnetic loops which come together at the flaring site. The flaring sites are generally assumed to be reconnection sites although observations have not establish a causal connection: "Direct evidence for reconnection in flares is difficult to find, despite the fact that it is thought to be the primary process behind flares" [1]. It is remarkable that often the post–flare loop systems begin to glow at the TRACE EUV wavelengths without substantial distortion: reconnection that probably took place appears to be (largely) completed by the time the loops are detected.

These observations pose a new challenge for the theories of quiescent as well as not so quiescent coronal structures and events. In this paper we examine the conjecture that the formation and primary heating of the coronal structures as well as the more violent events (possibly flares, erupting prominences and coronal mass ejections (CMEs)) are the expressions of different aspects of the same general global dynamics that operates in a given coronal region [4]. It is stipulated that the coronal structures are created from the evolution and re–organization of a relatively cold plasma flow emerging from the subcoronal region and interacting with the ambient solar magnetic field. The plasma flows, the source of both the particles and energy (part of which is converted to heat), in their interaction with the magnetic field, also become dynamic determinants of a wide variety of plasma states; it is likely that this interaction may be the cause of the immense diversity of the observed coronal structures [4–6]. Preliminary results from this magneto–fluid approach reproduce several of the salient observational features of the typical loops: the structure creation and primary heating are simultaneous – the heating takes place (by the viscous dissipation of the flow kinetic energy) in a few minutes, is quite non-uniform, and the base of the hot structure is hotter than the rest.

We plan to extend the scope of the magneto-fluid theory beyond the creation of the semi-quiescent coronal structures by seeking answers to the following: a) can the basic framework of this model predict the possibility of, and the pathways for the occurrence of sudden, eruptive, and catstrophic events (such as flares, eruptive prominences, CMEs) in the solar atmosphere, b) does the eventual fate, possibly catstrophic reorganization, of a given structure lie in the very conditions of its birth, c) is it possible to identify the range and relative values of identifiable physical quantities that make a given structure prone to eruption (flaring), d) will an eruption be the result of the conversion of excess magnetic energy into heat and bulk plasma motion as is generally believed to happen in the solar atmosphere [7]- [12] ?

We begin by identifying the quasi–equilibrium state of a typical coronal sturcture with a slowly changing Double–Beltrami (DB) state (one of the simplest, non–trivial magnetofluid equilibrium). The slow changes may be due to changes in the sun which affect the local magnetic fields, the interaction of various nearby structures, or disturbances in the solar atmosphere. The parameter change is assumed to be sufficiently slow that, at each stage, the system can find its local DB equilibrium (adiabatic evolution). The slow evolution must conserve the dynamical invariants: the helicity h_1, the generalized helicity h_2, and the total (magnetic plus the fluid) energy E. The problem of predicting sudden events (e.g. catastrophic eruption) then reduces to finding the range, if any, in which the slowly evolving structure may suffer a loss of equilibrium. The signature of the loss of equilibrium is quite easy to identify for the DB states. The transition may occur in one of the following two ways: 1) when the roots of the quadratic equation, determining the length scales for the field variation, go from being real to complex (implying change in the topology of the magnetic and the velocity fields — boundary separating the paramagnetic from the diamagnetic), or 2) the amplitude of either of the two states ceases to be real. For our current problem, the sudden change is likely to follow the second route.

By analysing a simple analytically tractable model, we find affirmative answers to all the four questions we posed. We show that the invariants h_1, h_2 , and E , which label and alongwith the initial and bounbary conditions determine the original state, hold the key to the eventual fate of a structure. If for a given equilibrium sequence, the total energy E is larger than some critical value (given in terms of h_1, and h_2), the catastrophic loss of equilibrium could certainly occur. The trigger for the equilibrium loss could come, for instance, from nearby structures getting close to each other with an increase in their interaction energy. The catastrophy pushes a DB state to relax to a minimum energy single Beltrami field. For coronal structures, the transition transfers almost all the short–scale magnetic energy to the flow energy.

II. MODEL

Within the framework of our approach, there are two distinct scenerios for eruptive events : a) when a "slowly" evolving structure finds itself in a state of no equilibrium, and b) when the process of creating a long–lived hot structure is prematurally aborted; the flow shrinks/distorts the structure which suddenly shines and/or releases energy or ejects particles. The latter mechanism requires a detailed time–dependent treatment and is not the subject matter of this paper. The following semi–equilibrium, collisionless magneto–fluid treatment pertains only to the former case.

A given structure is supposed to corresspond to the equilibrium solutions of the two–fluid system

$$\frac{\partial}{\partial t}\boldsymbol{\omega}_j - \nabla \times (\boldsymbol{U}_j \times \boldsymbol{\omega}_j) = 0 \quad (j = 1, 2) \tag{1}$$

written in terms of a pair of generalized vorticities $\boldsymbol{\omega}_1 = \boldsymbol{B}$, $\boldsymbol{\omega}_2 = \boldsymbol{B} + \nabla \times \boldsymbol{V}$, and effective flows $\boldsymbol{U}_1 = \boldsymbol{V} - \nabla \times \boldsymbol{B}$, $\boldsymbol{U}_2 = \boldsymbol{V}$, with the following normalizations: the magnetic field $\boldsymbol{B}$ to an appropriate measure of the magnetic field B_0, the fluid velocity $\boldsymbol{V}$ to the corresponding Alfvén speed, and the distances to the collisiolless ion skin depth l_i.

The simplest and perhaps the most fundamental equilibrium solution to (1) is given by the "Beltrami conditions", which imply the alignment of the vorticities and the corresponding flows ($\boldsymbol{\omega}_j // \boldsymbol{U}_j$),

$$\boldsymbol{B} = a(\boldsymbol{V} - \nabla \times \boldsymbol{B}), \tag{2}$$

$$\boldsymbol{B} + \nabla \times \boldsymbol{V} = b\boldsymbol{V}, \tag{3}$$

with a, $b = const.$ We have used constant density assumption for simplicity - extension to varying density is straightforward [4]. Equations (2) and (3) combine to yield:

$$(\mathrm{curl} - \lambda_+)(\mathrm{curl} - \lambda_-)\boldsymbol{B} = 0, \tag{4}$$

where $\tilde{a} = 1/a$, $\nabla\times$ = "curl", and

$$\lambda_\pm = \frac{1}{2}\left[(b - \tilde{a}) \pm \sqrt{(b + \tilde{a})^2 - 4}\right]. \tag{5}$$

For sub–alfvenic flows (the flows we generally encounter in the solar atmosphere), the length scales ($\lambda_\pm^{-1}$) are quite disparate. We assume $\lambda_+^{-1} >> \lambda_-^{-1}$ without loss of generality. The general solution to the "double Beltrami equations" (4) is a linear combination of the single Beltrami fields $\boldsymbol{G}_\pm$ satisfying $(\mathrm{curl} - \lambda)\boldsymbol{G} = 0$. Thus, for arbitrary constants $C_\pm$, the sum

$$\boldsymbol{B} = C_+\boldsymbol{G}_+ + C_-\boldsymbol{G}_- \tag{6}$$

solves (4), and the corresponding flow is given by $\boldsymbol{V} = (\lambda_+ + \tilde{a})\, C_+\boldsymbol{G}_+ + (\lambda_- + \tilde{a})\, C_-\boldsymbol{G}_-$.

The DB field encompasses a wide class of steady states of mathematical physics – from the force–free paramagnetic field to the fully diamagnetic field. The Beltrami conditions also demand "generalized Bernoulli conditions" which allow pressure confinement when an appropriate flow is driven [5] (and references therein).

The DB field is characterized by four parameters: λ_+, λ_- (eigenvalues), and C_+, C_- (amplitudes). The three invariants [13]: the helicity h_1, the generalized helicity h_2,

$$h_1 = \frac{1}{2}\int (\boldsymbol{A}\cdot\boldsymbol{B})\,\mathrm{d}\mathbf{r}, \qquad (7)$$

$$h_2 = \frac{1}{2}\int (\boldsymbol{A}+\boldsymbol{V})\cdot(\boldsymbol{B}+\nabla\times\boldsymbol{V})\,\mathrm{d}\mathbf{r}, \qquad (8)$$

($\boldsymbol{A}$ is the vector potential), and the total energy

$$E = \frac{1}{2}\int (\boldsymbol{B}^2+\boldsymbol{V}^2)\,\mathrm{d}\mathbf{r} \qquad (9)$$

will provide three algebraic relations connecting them [14]. To predict the possibility of an eruptive event, interpretted as the termination of an equilibrium sequence (for solar flares, this kind of an approach, albeit in different contexts, has been followed in numerous investigations, (see e.g. [15]- [16] and references therein), we analytically investigate this system using the macro–scale (λ_+^{-1}) of the closed strcuture as a control parameter. This choice is physically sensible and is motivated by observations because in the process of structure–structure interactions, "initial" shapes do undergo deformations/distortions with rates strongly dependent on the initial and boundary conditions.

For simplicity we explicitly work out the system in a Cartesian cube of length L. We take $\boldsymbol{G}_\pm$ to be the simple 2-D Beltrami ABC field [17],

$$\boldsymbol{G}_\pm = g_{x\pm}\begin{pmatrix}0\\ \sin\lambda_\pm x\\ \cos\lambda_\pm x\end{pmatrix} + g_{y\pm}\begin{pmatrix}\cos\lambda_\pm y\\ 0\\ \sin\lambda_\pm y\end{pmatrix}, \qquad (10)$$

with the normalization $(g_{x\pm})^2+(g_{y\pm})^2=1$. For real $\lambda_\pm$, (10) represents an arcade–magnetic field structure resembling interacting coronal loops [in Fig. 1]. Assuming $L = n_+(2\pi/\lambda_+) = n_-(2\pi/\lambda_-)$ ($n_\pm$ are integers), $\boldsymbol{G}_\pm$ satisfy the following relations: $\int \boldsymbol{G}_\pm^2 \mathrm{d}\mathbf{r} = L^2$, $\int \boldsymbol{G}_+\cdot\boldsymbol{G}_-\mathrm{d}\mathbf{r} = 0$, where $\int \mathrm{d}\mathbf{r} = \int_0^L\int_0^L \mathrm{d}x\mathrm{d}y$.

The invariants can now be readily evaluated and the results can be displayed in several equivalent forms. We find the following three equations to be the most convenient for further analysis ($h_2 = h_1 + \tilde{h}_2$, $\tilde{h}_2 = bE - \lambda_+\lambda_- h_1$):

$$\tilde{h}_2 = \frac{E}{2}\left[(\lambda_+ + \lambda_-) \pm \sqrt{(\lambda_+ - \lambda_-)^2 + 4}\right] - \lambda_+\lambda_- h_1, \qquad (11)$$

$$C_+^2 = D^{-1}\left\{E - \left[1+(\lambda_- + \tilde{a})^2\right]\lambda_- h_1\right\}\lambda_+, \qquad (12)$$

$$C_-^2 = -D^{-1}\left\{E - \left[1+(\lambda_+ + \tilde{a})^2\right]\lambda_+ h_1\right\}\lambda_-, \qquad (13)$$

where we have removed the common factor $L^2/2$, and

$$D = \left[1+(\lambda_+ + \tilde{a})^2\right]\lambda_+ - \left[1+(\lambda_- + \tilde{a})^2\right]\lambda_- = (\lambda_+ - \lambda_-)b(b+\tilde{a}). \qquad (14)$$

For given h_1, E, $\tilde{h}_2$ (h_2) and λ_+ (control parameter), we can solve the preceding system to determine the physical quantities λ_-, and $C_\pm$ which must all remain real for an equilibrium. Before we give an analytic derivation for the bifurcation conditions (leading to loss of equilibrium), we display in Fig.2 the plots of λ_- and $C_\pm$ as functions of λ_+ for two distinct sets for the values of the invariants: we choose $h_1 = 1$, $h_2 = 1.5$, $E = 0.4$ for Fig.2(a), and $h_1 = 1$, $h_2 = 1.5$, $E = 1.3$ for Fig.2(b) (dashed lines correspond to the region of imaginary C_-). We find that the behavior of the solution changes drastically with E. For the parameters of Fig.2(a), λ_- and $C_\pm$ remain real and change continuously with varying λ_+ implying that as the macroscopic scale of the structure ($1/\lambda_+$) changes continuously, the equilibrium expressed by (10) persists – there is no catastrophic or qualitative change. For Fig.2(b) with E changing from 0.4 to 1.3 (with same h_1, h_2) we arrive at a fundamentally different situation; when λ_+ exceeds a critical value $\lambda_+^{\mathrm{crit}}$, i.e, the macro–scale becomes smaller than a critical size, the physical determinants of the equilibrium cease to be real; the sequence of equilibria is terminated.

The condition for catasrophe turns out to be a constraint involving h_1, h_2 and E, which will allow the vanishing of C_-^2 for positive C_+^2, and real $\lambda_\pm$. It is straightforward to show that the system has a critical point if

$$E^2 \geq E_c^2 = 4\left(h_1 \pm \sqrt{h_1 h_2}\right)^2 \qquad (15)$$

and the critical λ is determined by a simultaneous solution of (11) and $E - \left[1+(\lambda_+ + \tilde{a})^2\right]\lambda_+ h_1 = 0$ giving:

$$\lambda_+^{\mathrm{crit}} = \frac{1}{2h_1}\left(E \pm \sqrt{E^2 - E_c^2}\right). \qquad (16)$$

Thus, for $E > E_c$ (determined by helicities h_1 and h_2), when the macroscopic size of a structure shrinks below a critical value, it can go through a severe reorganization.

At the critical point, an expected but most remarkable transition occurs. Using the value of $\lambda_+^{\mathrm{crit}}$, we find from equation(13) that the coefficient C_- , which measures the strength of the short scale fields, identically vanishes, and the equilibrium changes from Double Beltrami to a single Beltrami state defined by $\lambda_+ = \lambda_+^{\mathrm{crit}}$, i.e., $\boldsymbol{B} = C_+\boldsymbol{G}_+$ ($\nabla\times\boldsymbol{B} = \lambda_+\boldsymbol{B}$) with $\boldsymbol{V}$ parallel to $\boldsymbol{B}$. The transition leads to a magnetically more relaxed state with the magnetic energy reaching its minimum with appropriate gain in the flow kinetic energy (see Fig.3).

III. CONCLUSIONS

By modelling quasi–equilibrium, slowly evolving coronal structures as a sequence of Double–Beltrami magnetofiuid states in which the magnetic and the velocity field are self–consistently coupled, we have shown the possibility of, and derived the conditions for catastrophic changes leading to a fundamental transformation of the initial state. The critical condition comes out as an inequality involving three invariants of the collisionless magnetofluid dynamics. When the total energy exceeds a critical energy the DB equilibrium suddenly relaxes to a single Beltrami state corresponding to the large macroscopic size. All of the short–scale magnetic energy is lost having been transformed to the flow energy and partly to heat via the viscous dissipation of the flow energy.

This general mechanism in which the flows (and their interactions with the magnetic field) play an essential role could certainly help in advancing our understanding of a variety of sudden (violent) events in the solar atmosphere like the flares, the erupting prominences, and the coronal mass ejctions. The connection of flows with eruptive events is rather direct: it depends on their ability to deform (in specific cases distort) the ambient magnetic field lines to temporarily stretch (shrink, destroy) the closed field lines so that the flow can escape the local region with a considerable increase in kinetic energy in the form of heat/bulk motion.

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FIG. 1. Magnetic field line structure of a 2-D ABC map resembling coronal arcades.

FIG. 2. (a). Plots of λ_- and $C_\pm$ versus λ_+ for $E = 0.4 < E_c \simeq 0.45$, the critical energy. No catastrophe. (b). Plots of λ_- and $C_\pm$ versus λ_+ for $E = 1.3 > E_c$. There is a critical point at $\lambda_+ \simeq 0.041$.

FIG. 3. Plots for λ_-, the magnetic and the flow energies versus λ_+ for the catastrophe–prone set $h_1 = 1$, $h_2 = 1.007$ and $E = 1.3 > E_c = 7 \cdot 10^{-3}$. The scale lengths are highly separated $\lambda_+ << \lambda_-$. The initial choice makes $C_+ \sim O(\lambda_+/\lambda_-) \ll 1$ and $C_- \sim O(\lambda_-/\lambda_-) \sim 1$ from (12) and (13). If any interaction increases λ_+ (the size of the structure shrinks) the critical point ($\lambda_+ = \lambda_+^{\rm crit}$) will be reached at which C_- is zero. The magnetic field energy ($\propto C_+^2 + C_-^2$) decreases to a very small value since $C_+^2 \ll 1$. Since the total energy is conserved, almost all the initial magnetic energy is transfered to the flow causing an eruption. Notice that for coronal plasma, the skin depth l_i is small $\sim$ 100cm ($l_i/\lambda_+ \sim 10^3$km), for a typical density of $\sim 10^9\ {\rm cm}^{-3}$. A word of caution is necessary – as we approach the critical point, the quasiequilibrium considerations are just an indicator of what is happening – they must be replaced by a full time–dependent treatment to capture the dynamics; the changes are no longer slow.